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OPTIMAL GRASP PLANNING OF MULTI-FINGERED ROBOTIC HANDS: A REVIEW

WEI MIAO[1],GONGFA LI[1], GUOZHANG JIANG[1], YINFENG FANG[2], ZHAOJIE JU[2] AND HONGHAI LIU[2]

Abstract. This paper gives an overall review the optimal grasp planning of multi-fingered hands. In order to analyze multi-fingered grasp qualitatively, the contact models in common use are introduced and form closure and force closure are analyzed, then stable operation conditions of grasping are also proposed. This paper introduces three aspects of planning, which serves the purpose of how to make the optimal planning. The methods about the planning of grasping point location are presented, including geometric analysis based method, knowledge rules based method and optimization based method. The planning of grasping force is divided into optimization of grasping force in contact force space and optimization of grasping force in joint torque space. The planning of fingers gate is also analyzed. At the end of the paper, a few research challenges are highlighted and discussed.

Keywords: Multi-fingered hands; optimal grasp planning; form closure; force closure

AMS Subject Classification: 49K30

1. Introduction

At present, the research of multi-fingered robotic hands is becoming a hot spot in the field of robotics research. In structure and function, multi-fingered dexterous hand is more and more closed to the human hand. It solved the problems of single freedom paw so that the robots have better operation ability to accomplish complex assembly task and quickly adapt to different geometric objects grasping with the movement of the fingers. Grasp is a prerequisite for operation object to complete operational tasks. From the view of mechanism, grasp is a process that from the open chain to closed chain, from the independent movement to coordinate movement and from the no-load to the change of constraints and load.

Although the multi-finger dexterous hand optimized the performance of the industrial manipulator, its control and planning have become more complicated, therefore the grasp planning methods which can adapt to different geometric objects become the research focus of this kind of manipulator. Grasp planning is not only the basis of grasping stability, but also the prerequisite for dexterous manipulation. In order to simulate function of human hand better and complete a variety of grasp and operation tasks for the multi-finger dexterous hand, the human skills and experience should be treated formally and the planning should be reasonable for specific tasks [1].

[1] College of Machinery and Automation, Wuhan University of Science and Technology, Hubei, Wuhan, 430081, China, [2]Intelligent Systems and Biomedical Robotics Group, School of Computing, University of Portsmouth, Portsmouth PO1 3HE, United Kingdom,

e-mail: ligongfa@wust.edu.cn

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2. GRASPING MECHANISM OF MULTI-FINGERED HAND

2.1. **CONTACT MODEL.**

In the process of multi-fingered hands grasping manipulation, the fingers often cant apply force to the object in any direction. Generally, the contact models of the fingers and object have three basic forms, frictionless point contact, friction point contact and soft finger contact. When it's frictionless point contact, finger force only applied perpendicular to the contact surface; when it's friction point contact, fingers apply normal force and tangential force to contact surface; and the fingers not only apply normal force and tangential force to contact surface, but also a torque around the contact surface normal when it's soft contact.

In order to avoid slippage and separation on the contact point, the contact force must meet the following constraint conditions for the three types of contact models above:

Frictionless point contact:

$$f_{iz} > 0 \tag{1}$$

Friction point contact:

$$\sqrt{f_{ix}^2 + f_{iy}^2} \le \mu_1 f_{iz}, f_{iz} > 0 \tag{2}$$

Soft finger contact:

$$\sqrt{f_{ix}^2 + f_{iy}^2}/\mu_1 + |m_{iz}|/\mu_2 \le f_{iz}, f_{iz} > 0 \tag{3}$$

where f_{iz} is the normal force component on the contact point, f_{ix} and f_{iy} are tangential force component, m_{iz} is the normal force torque, μ_1 is the tangential friction coefficient, and μ_2 is the torque coefficient of soft finger contact.

2.2. **CLOSURE OF GRASPING.**

To ensure the stability of the grasping, the contact configuration formed by fingers and objects contact need to meet certain conditions to make the object keep force balance and have a ability of flexible operation object. This involves the analysis of the closure.

Form closure is that all the motion degrees of freedom of object are restricted by a group of frictionless one-way point contact constraints. Form closure method is that the contact constraints were regarded as the smooth contact and the rationality of the contact configuration could be evaluated according to whether the object motion degrees of freedom are zero. Form closure only relate to grasping objects geometric factors and grasping point position, and has nothing to do with friction. It is a simple geometric property.

Force closure means that any external force spiral which the objects suffered can be balanced by the force of fingers which meet the friction constraint conditions at the contact point. Force closure method is that the rationality of the contact configuration could be evaluated according to the force analysis of object and whether the object meets the static equilibrium conditions [2].

2.3. **OPERABILITY AND STABLE OPERATION CONDITIONS OF GRASPING.**

If the grasping can be achieved any movement of the object under the condition of keeping fingers always contact with object, it will be called grasp operability.

Stable operation conditions are: (1) grasping matrix $G \in R^{6\times n}$ is surjective, and joint torque vector point can generate the contact forces spiral in the interior of the friction cone at each contact point; (2) $R(G^T) \subset R(J)$ is that any contact velocity of the spiral $\dot{q} \in R^n$ can be generated by the joint velocity vector $\dot{q} \in R^m$ [3]. A grasping which meets above stable operating conditions can realize any given motion of the object and balance any external stress spiral under the condition of relative slip of the multi-fingered hand and object would not occur at the contact point [4].

Balanced grasp may be stable or unstable, and not all stable grasp meet force closure. The relationship between them is as shown in figure 1.

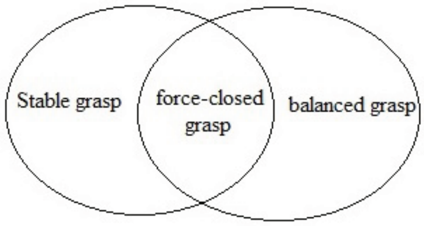

Figure 1. Relationship of stable, balanced and force-closed grasp

3. OPTIMAL GRASP PLANNING FOR MULTI-FINGERED HANDS

The grasp planning of multi-fingered hands is the fundamental problem of the dexterous hands operation, whose purpose is to determine the operation strategy of completing the expected task, such as grasp pattern, posture of hand and fingers, position of the contact points, grasping force and movement of finger joint. Grasp planning most currently focused on the optimal planning of contact points position, closed questions of grasping force and the planning of fingers gait.

3.1. **PLANNING OF CONTACT POINTS POSITION.**

The planning of contact points position is the main task of the grasp planning, which is to choose a set of optimal position of contact points on object according to some performance indicators. For the stable grasp, different layout of contact points has different stability degree. For planning the optimal contact position, international scholars mainly have put forward the following methods.

(1)Geometric analysis based method

The method of geometric analysis was the earliest research methods, which was used graphic method to form closure planning of two-dimensional grasp. This kind of method was assumed that grasp object is a rigid geometry.

Markenscoff [5] studied the closure property of grasping force problem using the method of geometry, and considered that at least four force screws were needed for the grasp of 2d flat object to ensure the grasping force closure. Nguyen [6] proposed an algorithm based on the shape of objects, which estimates the independence area of contact by force closure grasp so that the movement of objects is completely constrained. Borst et al. [7] developed an approach that to automatically determine the fourth point after already knowing the position of three points in the edge of the object based on the needs of operation task, which need not complex search iteration. Stappen et al. [8] put forward a form-closure grasp planning method to calculate the contact position of four fingers for the condition of frictionless grasping the polygon object. Li [9] proved force-closure algorithm of three fingers grasping using geometric method, which not only need simple calculation, but also can give the degree of stability.

(2) Knowledge rules based method

Grasp planning based on the knowledge rules was inspired by analyzing the human hand grasping. The effective mechanism of rule extraction could be set up through learning and saving the past grasp experience in this method. This kind of method can be further subdivided into the method based on the prototype, the method based on machine learning, the method based on expert system, the method based on data driven, etc.

Pollard [10] put forward integrated algorithm based on prototype. The prototypes are some common regular objects (such as spherical, cylindrical and conical, etc.), each of which is given some contact points. Grasp planner find out similar prototype according to grasp objects firstly, and then grasp the objects according to the grasp planning of prototype, so that reduce the computational time of integrated algorithm. But this method still exist some problems that need to be solved, for example, how to design prototype, how much the covering domain of prototype and how to express similarity, etc. Roa [11]-[12] discretized the contact surface, then searched for local optimal grasping position when changing one contact points every time starting from the initial grasping position. Stansfield [13] presented grasp planner based on the expert system, which obtain geometry information of the object using visual signals so that implement instantaneity. But this kind of the real-time grasping of unknown object is limited by visual technology and information processing speed, and is mainly used in two-dimensional space. Cutkosky [14] designed a grasping classification system based on expert system, which was regarded as a combination of the system grasping task, grasping objects and grasping tool, and can make corresponding decision for grasping in a variety of constraints. But due to various constraints, the optimal grasping models are hard to define. Coelho et al. [15] developed a grasping controller using the method of support vector machine, whose training used machine learning. But the simple polygon was only tested. In addition, data driven was also a research hotspot in recent years. For example, Elkoura [16] used inverse kinematics algorithm and human database established by human grasping posture to calculated joint position. Li and Pollard [17] captured the relevant data of people's actual grasping motion to build grasp database, then used shape matching algorithm to grasp strange objects. Akeotti [18] collected the practical grasp process using virtual reality technology to build database, so the system showed the good precision. But there were still problems with the system, for example, the adaptability of grasping new objects was not enough.

(3) Optimization based method

The optimization method is an optimization process which converts optimal grasping into an objective function. The properly objective function could be established using some grasp quality indexes.

Ferrari et al. [19] solved the problem of optimal force-closure grasp by working out the maximum sphere in force screw convex space. The advantage of this method is that the quality index is easy to calculate. Later, variety forms of quality indexes appeared. Lin et al. [20] developed a quality metric theory based on grasping stiffness matrices. The optimal grasping force could be calculated using elastic deformation energy equivalence principle. Li et al. [21] proposed evaluation basis of the minimum singular value of grasping matrix. Mirtich et al. [22] made a way of finding out the grasping position that offset the external disturbance force. And then the best grasping position would be optimized to offset the external torque. But this solution relied on the specific coordinate system. Ding et al. [23] regarded the problem of optimal contact position as a nonlinear programming problem. It assumed that some contact point positions were already known and the objective function would be minimized by choosing other contact positions.

Mo et al. [24] put forward a grasping method based on the maximum force spiral which was regarded as performance index and the optimization model of grasping position. The optimization model of grasping position and maximum force spiral were established under the condition of force-closure constraint. This method offset the limitation that generalized force ellipsoid is dimensionless so that express scraping effect clearly.

3.2. PLANNING OF GRASPING FORCE.

The analysis and planning of grasping force affect the quality of grasp. Dynamic force allocation was designed to find the optimal contact force that balance the dynamic force screw and meet contact constraint. For a given grasping configuration and known load object, grasping force should not only satisfy the force balance, but also meet the friction cone constraint generated by the contact of finger and the object.

The force balance equation of grasping can be expressed as:

$$Gf_c = -F_e \tag{4}$$

where $f_e \in R^n$ is the generalized contact force matrix in the contact friction cone, $G \in R^{6\times n}$is the grasping matrix, which only related to the layout of contact points on the object surface, $F \in R^6$is the generalized force vector acting on an object.

Corresponding to the contact force through your fingers, fingers joint torque determined by the following type:

$$J^T f_c = \tau \tag{5}$$

where $\tau \in R^m$ is the joint torque vector, $J \in R^{n\times m}$ is the Jacobean matrix of fingers.

Generally, the problem of grasping force could be related to the constraint optimization problem. Multi-finger hand grasping force can be divided into the following methods.

(1) Optimization of grasping force in contact force space

In this method, contact force and some indexes of contact force space were considered as the optimization variables and the objective function respectively. Then the optimized contact force map to the joint space to obtain the corresponding joint torque. The method can be divided into nonlinear optimization method, intelligent method, the linear optimization method.

Nakamura et al. [25] introduced the nonlinear constraint of friction cone, then the minimum norm of the friction was regarded as the optimization object function and the Lagrange multiplier method was used for nonlinear programming of force in the contact force space. Buss et al. [26] had turned the nonlinear constraint of friction cone into a linear constraint of a positive symmetric matrix, and calculated the optimal grasping force according to the Gradient flow theory. Bin Wang et al. [27] obtained initial grasping force, which was under the constraint of friction cone, by using the Lagrange multiplier method to automatically adjust the weight of contact force on each finger, and also used Gradient flow theory to optimize the initial grasping force. Cornella et al. [28] proposed a nonlinear programming algorithm based on the principle of duality. Borgstrom et al. [29] used penalty function to make the contact force satisfy with the constraint of friction cone. Jinbao Chen et al. [30] transferred the problem of precise grasping force optimization to the problem of nonlinear programming. For the solution of the problem, the fmincon function based on a sequential quadratic programming algorithm was put forward to efficiently solve the nonlinear programming problem and the grasping method of the contact force of each finger and contact force controlled by impedance control algorithm was used. This method was simple and autonomic.

Xiong [31] used the intelligent method of neural network for a real-time planning of grasping force. The planning goal was to minimize the contact force of fingertip. Constraint conditions included static friction constraint, one-way constraint and joint torque constraint.

In order to reduce the large amount of calculation and the retardation, scholars had proposed a linear programming method. Kerr and Roth [32] proposed a conservative algorithm for linear programming by turning the internal grasping force into linear programming problems of friction and joint torque constraints. Due to the linear programming methods in continuous system was lack of continuity, Sinha [33] established the optimization method to get the minimum force (or contact force) of nonlinear friction constraints by twice planning. Barkat et al. [34] also gave the twice planning algorithms that calculate the minimum pressure of contact point.

(2) Optimization of grasping force in joint torque space

In this method, the joint torques was directly taken as the optimization variable, and the optimization indexes of joint torque space were put forward as the objective function for optimization. Finally the optimal joint torque force would be obtained.

Qiuhao Zhang et al. [35] established the kinematics and dynamics of single branched robot, and solved the planning problem of joint torque that minimal joint torque was taken as the objective function and terminal motion trail was taken to constraint condition. Jiting Li et al. [36] transferred the friction constraint into the joint torque constraint based on the decomposition of contact force. The nonlinear joint torque was optimized using iteration algorithm with taking the joint torque as optimization variables and taking the maximization of relative carrying capacity of dexterous hand joint as the target function. Yu Guo and Zhijun Sun [37] built a nonlinear optimization model to determine the relationship between the contact force and the torques of multi-fingered hand joints with regarding corner position and the carrying capacity of the inner space of the multi-fingered hand joints as the optimization goal. Then the influence of the changes of direction in external spiral to contact force and contact safety margins got discussed.

3.3. **PLANNING OF FINGERS GAIT.**

In order to achieve the desired object movement, the core problem of the planning is how to determine the movement of the fingers. On the other hand, if the finger joint movement is known, the movement of objects could be predicted. It is a complex dynamic calculation process, which should ensure that fingers move in available space and there is no interference between the finger in the process of movement and the contact force keep balanced. Operating kinematics described the relationship of the movement of object, finger joints and contact point. Although the movements of object, joint and contact points exist at the same time, they have different status. The movement of finger joints is active and the movement of objects is expected, but the movement of the contact point is passive movement determined by the movement of finger joint and objects and the constraints of hand-object system [38]-[39]. According to the characteristics above, the rolling operation kinematics can be divided into two parts of instantaneous motion equation of finger joints and object and rolling contact equation.

Except for the contact between fingertips and object, multi-finger hand also grasp object through the knuckles and palms, so that the rank of multi-finger hands Jacobian matrix less than the number of contact constraint. In other word, the degree of the independence movement is less than the dimensions of the operating space and arbitrary space motion cannot achieved by the joint movement of multi-finger hand. For the hand-object system, Bicchi et al. [40] put forward an effective method to structure the kinematics equation.

In the process of the operation, expect for the movement of objects and finger joint, the movement of contact points on the contact surface of object and finger is also exist. So it is necessary to establish the relationship of relative motion between the contact surface and the movement of contact point. Cai and Montaon [41]-[43] made a thorough research on this issue. The basis kinematic equations in different forms were derived and relationship of relative motion between the contact surface and the movement of contact points was revealed.

4. CONCLUSION

The grasping operation of dexterous multi-finger hand is the emphasis and difficulty. It involves contact type, friction characteristics, the adaptability of grasping, stability, the feature modeling of grasping purpose, the quality modeling of grasping and so on. The primary challenge is that how the multi-fingered hands manipulate objects flexibly as human hands by learning from human hands or analyzing its movement [44]. So it is particularly important for the grasp planning of multi-finger hand. The problems of grasp planning have been extensively studied by scholars and a lot of planning methods and algorithms were put forward. The progress of multi-fingered grasping was presented and discussed in this paper. But because of the uncertainty of the environment and the multiplicity of grasping information, planning method need to be further improved.

(1) The parameterization of the surface of objects were required in most optimized grasping, which is very hard for irregular objects and most of the actual fingers layout is not taken into account by planning algorithm at present.

(2) The global optimal solution cannot be gotten by the optimal grasp planning, so a better grasp planning indicators needs to be put forward and develop more efficient optimization algorithm.

(3) At present, many corresponding methods were put forward from the grasping planning of personification grasping, machine learning, and geometric analysis and so on, but the problems were solved in different layers by different methods. Therefore, comprehensive methods of optimal grasp planning are needed in the future research.

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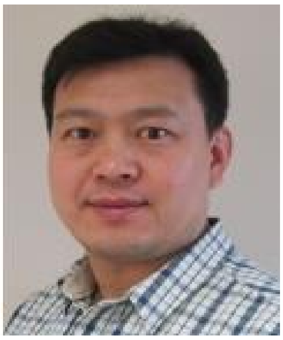
Wei Miao - he received B.S. degree in mechanical engineering and automation from Zhengzhou Huaxin College, Zhengzhou, China. He is currently occupied in his M.S. degree in mechanical design and theory at Wuhan University of Science and Technology. His current research interests include mechanical CAD/CAE, signal analysis and processing.

Gongfa Li - he received the B.S. degree, M.S. degree and Ph.D. degree in Wuhan University of Science and Technology, China. He is currently an associate professor in Wuhan University of Science and Technology. His major research interests are computer aided engineering, mechanical CAD/CAE, modelling and optimal control of complex industrial process.

Guozhang Jiang - he received the B.S. degree in Chang'an University, China. He received the M.S. degree in Wuhan University of Technology, China and Ph.D. degree in Wuhan University of Science and Technology, China. He is currently a professor in Wuhan University of Science and Technology. His research interests are computer aided engineering, mechanical CAD/CAE and industrial engineering and management system.

Yinfeng Fang - he is currently a Ph.D. candidate in the Intelligent Systems and Biomedical Robotics Group, School of Computing at University of Portsmouth.

Zhaojie Ju - he received the B.S. degree in automatic control and the M.S. degree in intelligent robotic both from the Huazhong university of Science and Technology, Hubei, China, in 2005 and 2007, respectively, and the Ph.D. degree in intelligent robotics at the University of Portsmouth, Portsmouth, U.K., in 2010. He is currently a lecturer in School of Computing, University of Portsmouth.. His research interests include machine intelligence, robot learning, pattern recognition and their applications in robots/prosthetic hand control, and human-robot interaction.



Honghai Liu - he received the Ph.D. degree in intelligent robotics from Kings College, University of London, London, U.K. He is currently a Professor of intelligent systems in Portsmouth University, Portsmouth, U.K. His research interests are approximate computation, pattern recognition, multi-sensor based information fusion and analytics, human machine systems, advanced control, intelligent robotics and their practical applications.